THE HISTORY OF THE PRIMALITY OF ONE—A SELECTION OF SOURCES

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The table below is a selection of sources which address the question “is the number one a prime number?” Whether or not one is prime is simply a matter of definition, but definitions follow use, context and tradition. Modern usage dictates that the number one be called a unit and not a prime.

We choose sources which made the author’s view clear. This is often difficult because of language and typographical barriers (which, when possible, we tried to reproduce for the primary sources below so that the reader could better understand the context). It is also difficult because few addressed the question explicitly. For example, Gauss does not even define prime in his pivotal *Disquisitiones Arithmeticae* [45], but his statement of the fundamental theorem of arithmetic makes his stand clear. Some (see, for example, V. A. Lebesgue and G. H. Hardy below) seemed ambivalent (or allowed it to depend on the context?)[1]

The first column, titled ‘prime,’ is yes when the author defined one to be prime. This is just a raw list of sources; for an evaluation of the history see our articles [17] and [110]. **Any date before 1200 is an approximation.** We would be glad to hear of significant additions or corrections to this list.

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[1]Context dependence is not unusual even today, for example when teaching mathematics, the function ‘log’ may represent the common (base 10) logarithm in a pre-calculus course, the multivalued inverse of the exponential in a complex variables course, and/or the single-valued real inverse of the exponential in a real analysis course.

prime	year	who	reference	quote (or comment)
no	400BC	Plato	[127, p. 276]	Tarán writes: "The Greeks generally, Plato and Aristotle included, considered two to be the first prime number (cf. Plato, *Republic* 524 D 7, *Parmenides* 143 C-144 A, pp. 14-15 *supra*, Aristotle, *Physics* 207 B 5-8, 220 A 27, *Metaphysics* 1016 B 17-20, 1021 A 12-13, 1052 B 20-24, 1053 A 27-30, 1057 A 3-6, 1088 A 4-8, Euclid, *Elem.* VlI, Defs. 1-2); and so for them one is not a number (Aristotle is explicit about this and refers to it as a generally accepted notion [cf. p. 20, note 95 and p. 35 with note 175]; for some late thinkers who treat one as an odd number cf. Cherniss, *Plutarch's Moralia*, vol. XIII, I, p. 269, n. *d*). Nor did the early Pythagoreans consider one to be a number, since in all probability they subscribed to the widespread notion that number is a collection of units (cf. Heath, *Euclid's Elements*, II, p. 280; Cherniss, *Crit. Pres. Philos.,* pp. 387 and 389)."
yes	350BC	Speusippus	[127, p. 276]	Tarán writes [127, p. 276] "Speusippus, then, is exceptional among pre-Hellenistic thinkers in that he considers one to be the first prime number. And Heath, *Hist. Gr. Math.*, I, pp. 69-70, followed by Ross, *Aristotle's Physics*, p. 604, and others, is mistaken when he contends that Chrysippus, who is said to have defined one as πλῆθος ἕν (cf. Iamblichus, *In Nicom. Introd. Arith.*, p. II, 8-9 [Pistelli]), was the first to treat one as a number (cf. further p. 38f. with note 189 *supra*)."
no	350BC	Aristotle	——	Heath says [61, p. 73] that "Aristotle speaks of the dyad as 'the only even number which is prime' (Arist. Topics, Θ. 2, 157 a 39). Also Tarán [127, p. 20] states Aristotle explicitly argues one is not a number (Metaphysics 1088 A 6-8), saying "Aristotle never considers one to be a number and for him the first number is two.". See also [127, p. 276].
no	300BC	Euclid	[61, p. 73]	Heath notes [61, p. 69] "Euclid implies [one is not a number] when he says that a unit is that by virtue of which each of existing things is called one, while a number is 'the multitude made up of units,' . . . " On p. 73 Heath mentions that Euclid includes 2 among the primes.
no	100BC	Theon of Smyrna	[121, p. 20]	Smith writes "Aristotle, Euclid, and Theon of Smyrna defined a prime number as a number 'measured by no number but an unit alone,' with slight variations of wording. Since unity was not considered as a number, it was frequently not mentioned. Iamblichus says that a prime number is also called 'odd times odd,' which of course is not our idea of such a number. Other names were used, such as 'euthymetric' and 'rectilinear,' but they made little impression upon standard writers. The name 'prime number' contested for supremacy with 'incomposite number' in the Middle Ages, Fibonacci (1202) using the latter but saying that others preferred the former."
			[61, p. 73]	Heath states Theon of Smyrna sees two as "odd-*like* without being prime" (Theon of Smyrna, p. 24. 7).

prime	year	who	reference	quote (or comment)
no	100	Nicomachus	——	"The Unit then is perfect potentially but not actually, for taking it into the sum as the first of the line I inspect it according to the formula to see what sort it is, and I find it to be prime and incomposite; for in very truth, not by participation like the others, but it is first of every number and the only incomposite" [66, p. 20]. Smith writes [121, p. 27] "It is not probable that Nicomachus (*c.* 100) intended to exclude unity from the number field in general, but only from the domain of polygonal numbers. It may have been a misinterpretation of the passage of Nicomachus that led Boethius to add the great authority of his name to the view that one is not a number." Tarán notes "For, if we started the number series with three (as some Neopythagoreans did [cf. e.g. Nicomachus, *Intr. Arith.* I, II], who consider prime number to be a property of odd number only [cf. Tarán, *Asclepius on Nicomachus*, pp. 77-78, on I, νη and ξα, with references]), then there would be in ten three prime numbers (3, 5, 7) and five composite ones (4, 6, 8, 9, 10)." Heath notes that Nicomachus defines primes, composites subdivisions of the odds [61, p. 73], so two is not prime. Also "According to Nicomachus 3 is the first prime number..." [60, p. 285]
—	300	Iamblichus	[61, p. 73]	Heath notes that Iamblichus defines primes and composites as subdivisions of the odds, so two is not prime.
no	400	Martianus Capella	[124, pp. 285–286]	"[743] We have briefly discussed the numbers comprising the first series, the deities assigned to them, and the virtues of each number. I shall now briefly indicate the nature of number itself, what relations numbers bear to each other, and what forms they represent. A number is a collection of monads or a multiple preceeding from a monad and returning to a monad. There are four classes of integers: the first is called 'even times even'; the second 'odd times even'; the third 'even times odd'; and the fourth 'odd times odd'; these I shall discuss later. [744] Numbers are called prime which can be divided by no number; they are seen to be not 'divisible' by the monad but 'composed' of it: take, for example, the numbers five, seven, eleven, thirteen, seventeen, and others like them. No number can divide these numbers into integers. So they are called 'prime,' since they arise from no number and are not divisible into equal portions. Arising in themselves, they beget other numbers from themselves, since even numbers are begotten from odd numbers, but an odd number cannot be begotten from even numbers. Therefore prime numbers must of necessity be regarded as beautiful.

prime	year	who	reference	quote (or comment)
				[745] Let us consider all numbers of the first series according to the above classifications: the monad is not a number; the dyad is an even number; the triad is a prime number, both in order and in properties; the tetrad belongs in the even times even class; the pentad is prime; the hexad belongs to the odd times even or even times odd (hence it is called perfect); the heptad is prime; the octad belongs to . . . ”
no	500	Boethius	[90, pp. 89-95]	Like Nicomachus, defines prime as a subdivision of the odds, and starts his list of examples at 3.
no	550	Cassiodorus	[51, p. 5]	A prime number “is one which can be divided by unity alone; for example, 3, 5, 7, 11, 13, 17, and the like.” For him, prime is a subset of odd; perfect, abundant and deficient are all subsets of even [67, pp. 181-182].
no	636	Isidore of Seville	[51, pp. 4-5]	In “Etymologiarum sive Originum, Liber III: De mathematica” Isidore says (Grant’s translation[2]): “Number is a multitude made up of units. For one is the seed of number but not number. . . . Number is divided into even and odd. Even number is divided into the following: evenly even, evenly uneven, unevenly even and unevenly uneven. Odd number is divided into the following: prime and incomposite, composite, and a third intermediate class *(mediocris)* which in a certain way is prime and incomposite but in another way secondary and composite. . . . Simple [or prime] numbers are those which have no other part [or factor] except unity alone, as three has only a third, five only a fifth, seven only a seventh, for these have only one factor.”
no	825	Al-Khwarizmi	[106, p. 812]	“Boetius (AD 475–524/525), who wrote the most influential book of mathematics during the Middle Ages, *De Institutione Arithmetica Libri Duo*, following a personal restrictive interpretation of Nichomachus and affirmed that one is not a number. Even Arab mathematicians (e.g. Abu Ja’far Mohammed ibn Musa AI-Khowarizmi, *c.* AD 825) excluded unity from the number field. Rabbi ben Ezra (*c.* 1095–*ca.* 1167), instead in his *Sefer ha-Echad* (Book of Unity) argued that one should be looked upon as a number. Only during the 16th century did authors begin to raise the question as to whether this exclusion of unity from the number field was not a trivial dispute (Petrus Ramus, 1515–1572), but Simon Stevin (*c.* 1548–*c.* 1620) argued that a part is of the same nature as the whole, and hence, that unity is a number.”

[2]There is a wonderful 1493 version of this text online `http://tudigit.ulb.tu-darmstadt.de/show/inc-v-1/0039`.

prime	year	who	reference	quote (or comment)
no	850	al-Kindī	[68, p. 102]	After considering and rejecting the possibility of one being a number al-Kindī writes: "Since, therefore, it is clear that one is not a number, the definition said of number shall then encompass /number fully, *viz.*, that it is a magnitude (composed of) onenesses, a totality of onenesses, and a collection of onenesses. Two is, then, the first number." (He did see the number two "as prime, if in a qualified way" [68, p. 181].)
no	1120	Hugh of St. Victor	[51, p. 56]	"Arithmetic has for its subject equal, or even, number and unequal, or odd, number. Equal number is of three kinds: equally equal, equally unequal, and unequally equal. Unequal number, too, has three varieties: the first consists of numbers which are prime and incomposite; the second consists of numbers which are secondary and composite; the third consists of numbers which, when considered in themselves, are secondary and composite, but which, when one compares them with other numbers [to find a common factor or denominator], are prime and incomposite."
—	1140	Rabbi ben Ezra	[121, p. 27]	Smith notes "One writer, Rabbi ben Erza (*c.* 1140), seems, however, to have approached the modern idea. In his *Sefer ha-Echad (Book on Unity)* there are several passages in which he argues that one should be looked upon as a number." On the other hand, M. Friedländer [43, p. 658] notes "The book [Abraham Ibn Ezra's Arithmetic] opens with a parallelism between the Universe and the numbers; there we have nine spheres and a being that is the beginning and source of all the spheres, and at the same time separate and different from the spheres. Similarly there are nine numbers, and a unit that is the foundation of all numbers but is itself no number.
—	1202	Fibonacci	*Liber Abaci*	Smith quotes Fibonacci as follows [121, p. 20]:"Nvmerorum quidam sunt incompositi, et sunt illi qui in arismetrica et in geometria primi appellantur. . . . Arabes ipsos hasam appellant. Greci coris canon, nos autem sine regulis eos appellamus [*Liber Abaci*, I, 30]." Besides Fibonacci's preferred 'incomposite,' 'simple number' also seems common in the later periods. See also the 1857 copy of *Liber Abaci* [83, p. 30].
no	1483	Prosdocimo	[122, pp. 13-14]	"It follows [Euclid and] 'Bohectius' (Boethius) in defining number and in considering unity as not itself a number, as is seen in the facsimile of the first page [Fig. 6, pg 14]."

prime	year	who	reference	quote (or comment)
—	—	——	——	L. L. Jackson, writing about the teaching of mathematics in the sixteenth century, notes [65, p. 30]: "This difference of opinion as to the nature of unity was not new in the sixteenth century. The definition had puzzled the wise men of antiquity. Many Greek, Arabian, and Hindu writers had excluded unity from the list of numbers. But, perhaps, the chief reason for the general rejection of unity as a number by the arithmeticians of the Renaissance was the misinterpretation of Boethius's arithmetic. Nicomachus (c. 100 A.D.) in his $A\rho\iota\theta\mu\eta\tau\iota\kappa\eta\varsigma\ \beta\iota\beta\lambda\iota\alpha\ \delta\nu o$ had said that unity was not a polygonal number and Boethius's translation was supposed to say that unity was not a number. Even as late as 1634 Stevinus found it necessary to correct this popular error and explained it thus : $3 - 1 = 2$, hence 1 is a number.'
no	1488	John of Holywood	[125, p. 47]	"Therfor sithen þe ledynge of vnyte in hym-self ones or twies nought comethe but vnytes, Seithe Boice in Arsemetrike, that vnyte potencially is al nombre, and none in act. And vndirstonde wele also that betwix euery." The editor noted beside this section that "Unity is not a number."
no	1526	P. Ciruelo	[26, p. 15]	Primes are a subset of the odds: "Numeri imparis tres ſunt ſpecies immediate quæ ſunt, primus, ſecundus, & ad alterum primus. Numerus impar primus eſt qui ſola vnitate parte aliquota metiri poteſt, vt. 3. 5. 7. idemꝗ incompoſitus nominatur, & ratio vtriuſꝗ denominationis eſt eadem : quia numeri imparis nulla poteſt eſſe pars aliquota præter vnitatem, niſi illa etiam ſit numerus impar." This source did not have page numbers, but this quote is on the 15th page.
no	1537	J. Köbel	[93, p. 20]	"Wherefrom thou understandest that 1 is no number / but it is a generatrix / beginning / and foundation of all other numbers." [also gives the original] "*Darauss verstehstu das I. kein zal ist / sonder es ist ein gebererin / anfang / vnd fundament aller anderer zalen.*"
no	1561	G. Zarlino	[138, p. 22]	[Music book] "*Li numeri Primi & incompoſti ſono quelli, i quali non poſſono eſſer numerati o diuiſi da altro numero, che dall' vnita; come, 2. 3. 5. 7. 11. 13. 17. 19. & altri ſimili*"
—	1585	M. Stevin	——	"Michael[3] Stevin . . . was probably the first mathematician expressly to assert (in 1585) the numerical nature of One" [93, p. 20]. However, there are others. First might be Speusippus (ca. 365BC) [127, pp. 264, 276], but these folks seem to be rare and had little effect on common thought. Speusippus viewed one as prime.
no	1603	P. A. Cataldi	[19, p. 40]	List of primes starts at 2.

[3]We are not sure why Menninger calls him Michael Stevin instead of Simon Stevin, but the context makes it clear of whom he is speaking.

prime	year	who	reference	quote (or comment)
no	1611	C. Clavius	[27, p. 307]	[Commentary on Euclid] “PRIMVS numerus eſt, quem vnitas ſola metitur. *QVOD ſi numerum quempiam nullus numerus, ſed ſola vnitas metiatur, it a vt neg, pariter par, neg, pariter impar, neque impariter impar poßit dui, appellabitur numerus primus; quales ſunt omnes iſti 2. 3. 5. 7. 11. 13. 17. 19. 23. 29. 31. &c. Nam eos ſola vnitas metitur.*”
no	1615	D. Henrion	[37, p. 207]	[Paraphrased Euclid] “11. Nombre premier, eſt celuy qui eſt meſuré par la ſeule vnité. *C’eſt à dire, que ſi vn nombre n’eſt meſuré par aucun autre nombre, mais ſeulement par l’vnité, il eſt nombre premier, & tels ſont tous ceux-cy* 2.3.5.7.11.13.17.19.23.29.31. *&c. Car la ſeule vnité meſure iceux.*” This was very slightly reworded in a later 1676 edition [38, p. 381] (after his death): “11. Nombre premier, eſt celuy qui eſt meſuré par la ſeule unité. *C’eſt-à-dire, que ſi un nombre n’eſt meſuré par aucun autre nombre, mais ſeulement par ſoy même, & l’unité, il eſt nombre premier, & tels ſont tous ceux-cy* 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, *&c. Car chacun d’iceux n’eſt meſuré par aucun autre nombre, mais par la ſeule unité.”* (Note that Denis Henrion and Pierre Hérigone are *both* pseudonyms for the Baron Clément Cyriaque de Mangin (1580–1643).[4])
no	1625	M. Mersenne	[94, pp. 298-299]	“*Les nombres premiers entr’eus ſont ceus qui ont la ſeule unité pour leur meſure commune : & les nombres compoſez ſont ceux qui ſont meſurez par quelque nombre, qui leur ſert de meſure commune.* Ce Thorême comprend la 13. & 14. definition du 7, & n’a beſoin que d’explication: ie di donc premierementque le nõbre premier n’a autre meſure que l’ vnité, tel qu’eſt, 2, 3, 5 &c. vous treuuerez les autres nombres premiers par l’ordre naturel des nõbres *impairs*, ſi vous en oſtez tous les nõbres qui ſont éloignez par 3. nombres du 3, & par cinq nombres du 5, & par 7, nombres du 7, & ainſi des autres, ...” Another example: “... il faut multiplier tous les nombres premiers moindres que 10, a scauoir 2, 3, 5, 7.” [95, p. 23]
no	1640	A. Metius	[96, pp. 43-44]	“Numeri conſiderantur aut abſolutè pe ſe : aut inter ſe relativè. Numerus abſolutè pe ſe conſideratus, eſt aut per ſe Primus, aut Compoſitus. Numerus per ſe primus eſt, quem præter unitatem nullus alius numerus metitur, *quales ſunt 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, &c. namque eos ſola unitas dividit, ut nihil ſuperſit.*”
no	1642	M. Bettini	[11, p. 36]	[Euclid’s] “Qui lib. 7 def. 11 ſic: *Primus numerus eſt, quem vnitas ſola metitur*, quales ſunt. 2. 3. 5. 7. 11. 13. 17. 19. 23. 29. 31. &c. primos numeros & inuenire, & infinitos eſſe docet lib. 9. propoſ. 20.”

[4] http://www-history.mcs.st-andrews.ac.uk/Biographies/Herigone.html

prime	year	who	reference	quote (or comment)
no	1657	Léon de Saint-Jean	[116, p. 581]	"Sunt infuper numeri *Primi*, qui fola vnitate, nec alio præter vnitatem numero, menfurantur. Dicitur autem numerus vnus alterum menfurare, qui multoties repetitus alterum ita explet ; vt nihil fuperfluat, aut defit. Itaque vocantur numeri primi ac *Simplices*, quales funt 2. 3. 5. 7. 11. 1. 3.[*sic*] 17. 19. 23 &c."
no	1657	F. v. Schooten	[118, pp. 393-403]	[primes only used to find divisors] His "Sectio V. *Syllabus numerorum primorum, qui continentur in decem prioribus chiliadibus.*" List of primes begins with 2 (p. 394).
yes	1668	Brancker & Pell	[89, p. 367]	[Table] "It may be of great ufe fometimes to have *a complete and orderly enumeration of all incompofits between 0, and 100,000, without any mixture of Compofits*; thus 1. 2. 3. 5. 7. 11. 13. &c, leaving out 9, 21 and all other compofits." [108, p. 201]. Maseres reprints the appendix from *Teutsche Algebra* which contains the tables (see [89, Preface p. vii, 353]) as pages 353 to 416 of his text. Bullynck states that "The authorship of this book [Brancker's translation of Rahn's *Teutsche Algebra*] has been a matter of debate, but it is by now certain that Rahn was a student of Pell in Zürich and mainly used Pell's lectures to write his book. Already in 1668 the book has therefore been known as Pell's *Algebra*, and the *Table of Incomposits* has likewise been known as Pell's Table, though Keller and Brancker, independently, calculated it" [14].
no	1673	S. Morland	[100, p. 25]	[Euclid] "A prime *number* is that which is meafured onely by an Unite. That is to fay 2, 5, 7, 11, 13, *&c* are *prime numbers*, becaufe neither of them can poffibly be divided into equal parts by any thing lefs then an Unite." (He did not include 3 in his list of primes.)
no	1679	J. Moxon	[101]	[Euclid] (Defines Number on p. 97, Primes on p. 118, and Unity on p. 162) "**Prime**, or **First Number**, Is defined by *Euclid* to be that which onely Unity doth meafure, as 2, 3, 5, 7, 11, 17, 19, 33, 29, 31, &c. for onely Unity can meafure thefe. [101, p. 118]; "**Number**, Is commonly defined to be, *A Collection of Units*, or *Multitude compofed of Units*; fo that *One* cannot be properly termed a *Number*, but the begining of *Number*: Yet I confefs this (though generally received) to fome feems queftionable, for againft it thus one might argue: A Part is of the fame matter of which is its Whole; An Unit is part of a multitude of Units; Therefore an Unit is of the fame matter with a multitude of Units: But the matter and fubftance of Units is Number; Therefore the matter of an Unit is Number. Or thus, A Number being given, If from the fame we fubtract 0, (no Number) the Number given doth remain: Let 3 be the Number given, and from the fame be taken 1, or an Unit, (which, as thefe will fay, is no Number) then the Number given doth remain, that is to fay, 3, which to fay, if abfurd. But this by the by, and with fubmiffion to better Judgments." [101, p. 97]

prime	year	who	reference	quote (or comment)
no	1680	V. Giordani	[39, p. 310]	"*Tuttii numeri, che non poſſono eſſere miſurati giuſtamente da altri numeri, cioe che non ſono numeri parimente pari, ne parimente diſpari, ne meno diſparimente diſpari, mà che poſſono eſſere miſurati ſolamente dall'vnità, ſi dicono numeri primi: come ſono i ſeguenti 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41 &c.*"
no	1682	G. Clerke	[29, p. 39]	"Docuit *Euclides*, lib. 7. Definit. 11. numerum illum eſſe primum quem unitas ſola metitur, hoc eſt, dividit, ita, 2 3 5 7 11 13 17 19 23 29 31, &c. ſunt omnes primi :. . . "
yes	1685	F. Wallis	[89, p. 292]	(Wallis' work is reprinted as pp. 269-352 of [89]; also repeated in [130, p. 496]) "6. It is manifeſt that the Number 1, hath no Aliquot Part, and but one Diviſor, that is 1. Becauſe there is no Number leſs than itſelf that may be a part of it : But it meaſures itſelf ; and therefore is its own Diviſor. 7. Any other Prime Number hath one Aliquot Part, and Two Diviſors. . . . 8. Every *Power* of a *Prime Number* (other than of 1, which here is underſtood to be excluded,) . . . "
no	1685	T. Corneille	[32, p. 110]	[Dictionary] "NOMBRE. ſ.m. *Pluſieurs unites conſiderées enſemble* . . . *Nombre premier*, Celuy que la ſeule unité meſure; comme 2. 3. 5. 7. 11. qu'on ne ſçauroit meſurer par aucun autre nombre, . . . "
yes	1689	J. Prestet	[107, p. 141]	"Je nommerai *nombres ſimples* ou *premiers*, ceux qu'on ne peut diviſer au juſte ou ſans reſte par aucun autre entier que par eux-mêmes ou par l'unité; comme chacun des dix 1, 2, 3, 5, 7, 11, 13, 17, 19, 23."
no	1690	C. Chales	[21, p. 169]	[Expounding Euclid book 7] "1. Unitas eſt ſecundùm quam unumquodque dicitur unum. Nempe ab unitate dicitur unus homo, unus leo, unus lapis. Hæc definitio dat primam tantum unitatis cognitionem, quod in præſenti materia ſufficit, unitatem enim per ſe melius cognoſcimus, quàm ex quacumque definitione. 2. Numerus eſt ex unitatibus compoſita multitudo. *Unde tot habet partes quot unitates, denominationemque habet ex multitudine unitatum. Ex quo ſequitur omnes numeros inter ſe commenſurabiles eſſe, cum eos unitas metiatur.* 11. Primus numerus abſolutè dicitur is quem ſola unitas metitur, *ut 2, 3. 5. 7. 11. 13, quia nullam habent partem aliquotam unitate majorem.*"
no	1690	A. Arnauld	[3, p. 98]	"On dit qu'un nombre eſt nombre premier, quand il n'a de meſure que l'unité & ſoy-même, (ce qui ſe ſous-entend ſans qu'on le diſe.) Comme 2. 3. 5. 7. 11. 13, &c."
no	1691	J. Ozanam	[103, p. 27]	[Expositor] "Le *Nombre Premier* eſt celuy qui n'eſt meſuré par aucun nombre que par l'unité: comme 2, 3, 5, 7, 11, 17, 19, &c. On le nomme auſſi *Nombre lineaire*, & encore *Nombre incompoſé*, pour le differencier du *Nombre compoſé*." [Where is 13?]

prime	year	who	reference	quote (or comment)
—	—	——	——	Ağargün and Özkan, in "A historical survey of the fundamental theorem of arithmetic" [1] address the development of the fundamental theorem of arithmetic and affirm with C. Goldstein [49] that up to the 17th century they were not interested in the prime factorization integers for its own sake, but as a means of finding divisors. Note how this may alter the way you view the primality of one...
no	1720	E. Phillips	[105, p. 460]	"**Prime**, **Simple**, or **Incompofit Number**, (in *Arithm.*) is a Number, which can only be meafur'd or divided by it felf, or by Unity, without leaving any Remainder; as 2, 3, 5, 7, 11, 13, *&c.* are Prime Numbers. **Compofite** or **Compound Number**, is that which may be divided by fome Number, lefs than the Compofite it felf, but greater than Unity; as 4,6, 8, 9, 10, &c." (This book does not have page numbers but this is on the 460th page.)
no	c.1720	"Shuli Jingyun"	[113]	Denis Roegel reconstructed the tables from the Siku Quanshu (c.1782) *which are supposedly* copies of those from the Shuli Jingyun (1713-1723) [113]. The list of primes begins $2, 3, 5, 7, \ldots$ [5] Again: from the Siku Quanshu (c.1782).
yes	1723	J. Harris	[59]	[Table] Included Brancker and Pell's Table of Incomposits
no	1723	F. Brunot	[13, p. 3]	"*Le Nombre premier, ſimple,* ou *qui n'eſt pas compoſé*, eſt celui qui n'a aucunes parties aliquotes que l'unité, comme 2, 3, 5, 7, 11, 13, &c." [Defines primes first, then composites]
no	1724	J. Cortès	[33, p. 7]	[States that he follows Euclid on a previous page] "El Numero primero ſe dize aquel que de ſola la unidad puede ſer medido, y no de otro numero, como 2. 3. 5. 7. 11. 13. y otros de eſta manera."
no	1726	E. Stone	[126, p. 293]	[Dictionary] "PRIME NUMBERS, in Arithmetick, are thofe made only by Addition, or the Collection of Unites, and not by Multiplication : So an Unite only can meafure it; as 2, 3, 4, 5, *&c.* and is by fome call'd a *Simple*, and by others an *Uncompound Number*." (This book does not contain any page numbers but this is on the 293rd page.)
no	1728	E. Chambers	[22]	"PRIME *Number*, in Arithmetic, a Number which can only be meafur'd by Unity; or whereof 1 is the only aliquot part. ... Such are 5, 7, 11, 13, *&c.*" [22, p. 871] "'Tis difputed among Mathematicians, whether or no *Unity* be a *Number*.—The generality of Authors hold the Negative; and make *Unity* to be only inceptive of Number, or the Principle thereof; as a Point is of Magnitude, and *Uniſon* of Concord. *Stevinus* is very angry with the Maintainers of this Opinion : and yet, if Number be defin'd a Multitude of *Unites* join'd together, as many Authors define it, 'tis evident *Unity* is not a Number." [22, p. 323]

[5] Original image `http://www.archive.org/stream/06076320.cn#page/n66/mode/2up`.

prime	year	who	reference	quote (or comment)
no	1735	J. Kirkby	[69, p. 7]	"53. An *Even Number* is that which is meafured by 2. 54. An *Odd Number* is one more than an even Number. 55. A *Prime* or *Incompofite Number* is that which no Number meafures but Unity, as 3, 5, 7, 11, 13, 17, 19."
no	1739	C. R. Reyneau	[112, p. 248]	"On remarquera fur les nombres que leurs divifeurs premiers ne font pas toujours de fuite les nombres premiers 2, 3, 5, 7, 11, *&c*."
yes	1742	C. Goldbach	[48]	Letter to Euler with the "Goldbach Conjecture"
yes	1746	G. S. Krüger	[73, p. 839]	Prime list starts with the number 1.
yes	1759	M. L. Willich	[137, p. 831]	Factor list starts "1 primi."
yes	1762	N. Caille & K. Scherffer	[16, p. 13]	"Numerus, qui nullius alterius, quam unitatis, eft multiplus, dicitur *numerus primus*. Horum numerorum amplæ tablæ apud varios fcriptores extant; en eos, qui centenario funt inferiores: 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97,"
no	1770	L. Euler	[40, pp. 14–16]	"But, on the other hand, the numbers 2, 3, 5, 7, 11, 13, 17, &c. cannot be represented in the same manner by factors, unless for that purpose we make use of unity, and represent 2, for instance, by 1×2. But the numbers which are multiplied by 1 remaining the same, it is not proper to reckon unity as a factor. All numbers, therefore, such as 2, 3, 5, 7, 11, 13, 17, &c. which cannot be represented by factors, are called *simple*, or *prime numbers*; whereas others, as 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, &c. which may be represented by factors are called *composite numbers*. (This is followed by a nice "we can trace no regular order [in the primes]" quote.
yes	1770	J. H. Lambert	[74, p. 73]	[Tables] Table VI, Numeri Primi begins with 1, 2, 3, 5, 7, 11, ...; repeated in the Latin version (same table number and page) [75, p. 73]
no	1772	S. Horsley	[64, p. 332]	"Hence it follows, that all the Prime numbers, except the number 2, are included in the feries of odd numbers, in their natural order, infinitely extended; that is, in the feries 3. 5. 7. 9. 11. 13. 15.. . . "
yes	1776	A. Felkel	[41, p. 16]	Table of primes begins 1, 2, 3, 5, 7, 11, ...
yes	1782	E. Waring	[131, p. 379], [133, p. 362b]	"1. Omnis par numerus conftat e duobus primis numeris,. . . " [Every even number is the sum of two primes, . . .]. "3. . . . hic excipiantur duæ arithmeticæ feries [of prime numbers] 1, 2, 3 & 1, 3, 5, 7." (He is explaining the only two arithmetic sequences of primes which do not have a common difference divisible by 6.) Also [132, p. 391] ". . . adding the prime numbers 1, 2, 3, 5, 7, 11, 13, 19, &c. . . . " [Where is 17?]

prime	year	who	reference	quote (or comment)
yes	1785	A. G. Rosell	[114, p. 39]	"De este modo, 1, 2, 3, 5, 7, 11, 13, &c. son números primeros, y 4, 6, 8, 9, 10, &c. números compuestos."
yes	1786	A. Bürja	[15, p. 45]	"Eine Primzahl oder einfache Zahl nennet man diejenige die durch keine andere, ſondern nur allein durch die Einheit und durch ſich ſelbſt gemeſſen wird. Z. E. 1, 2, 3, 5, 7, 11, 13, 17 ſind Primzahlen. Daß aber jede Zahl durch die Einheit und durch ſich ſelbſt gemeſſen wird, bedarf keineſ Beweiſes." [Nice cover]
no	1789	F. Meinert	[92, p. 69]	"So ſind 2, 3, 5, 7, 11, 13 u. Primzahlen; 4, 6, 8, u. aber zuſammengeſetzte Zahlen."
no	1801	C. F. Gauss	[45]	Gauss states and proves (for the first time) the uniqueness case of the fundamental theorem of arithmetic: "A composite number can be resolved into prime factors in only one way" [45]. Euler (1770) assumed and Legendre (1798) proved the existence part of this theorem [1]. (Preset (1689) used, and al-Fārisī (ca. 1320) may have also proved, the existence part of this theorem.) Gauss' table had 168 primes below 1000 in [46, p. 436] (including 1 as prime would give 169).
yes	1807	A. M. Chmel	[24, p. 65]	"Numerus integer praeter se ipſum et unitatem nullum alium diviſorem (menſuram) hacens, dicitur *ſimplex*, vel *numerus primus*, (Primzahl). Numerns autem talis, qui praeter ſe ipſum et 1, adhuc unum vel plures diviſores habet, vocatur *compoſitus*. *Coroll*. 1. Numeri *primi* ſunt: 1, 2, 3, 5, 7, 11, 13, 17, 19 etc. *Compoſiti*: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18 etc."
no	1808	G. S. Klügel	[70, p. 892]	"Primzahl, einfache Zahl, (numerus primus) iſt eine ſolche, welche keine ganze Zahlen ju Factoren hat oder, welche nur von der Einheit allein gemeſſen wird, wie die Zahlen 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 u. ſ. f."
no	1811	P. Barlow	[6, p. 54]	"2 3 5 7 . . . 97, which are all of the prime numbers under 100." Again in 1847: "A *prime number* is that which cannot be produced by the multiplication of any integers, factors, or that cannot be divided into any equal integral parts greater than unity." [7, p. 642].
yes	1818	J. G. Garnier	[44, p. 86]	"*Lambert*, et tout récemment l'astronome *Burkardt* ont donné des tables très-étendues de nombres premiers qui servent à la décomposition d'un nombre en ses facteurs nombres premiers." [Then he gives a table with primes, starting at 1, extending to 500.]
yes	1825	O. Gregory	[52, pp. 44-45], [53, pp. 40-42]	1. A *unit*, or *unity*, is the representation of any thing considered individually, without regard to the parts of which it is composed. 2. An *integer* is either a unit or an assemblage of units : and a *fraction* is any part or parts of a unit. . . . 4. One number is said to *measure* another, when it divides it without leaving any remainder. . . . 8. A *prime number*, is that which can only be measured by 1, or unity. [On the next page he lists the first twenty primes starting with 1. Second reference is German]

prime	year	who	reference	quote (or comment)
yes	1830	A. M. Legendre	[81, p. 14]	Presenting Euclid's argument there are infinitely many primes, he begins "Car si la suite des nombres premiers 1.2.3.5.7.11, etc. était finie, et que p fût le dernier ou le plus grand de tous, ..."
yes	1832	F. Minsinger	[99, pp. 36–37]	[Schoolbook] Lists 169 prime from 1 to 1000 (starting with 1)
no	1834	M. Ohm	[102, p. 140]	"**Note: Die erstern Primzahlen sind der Reihe nach:** 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, [...]" [Mathematician, the Physicist's brother]
no	1835	A. Reynaud	[111, pp. 48–49]	"*Un nombre est dit* PREMIER, *lorsqu'il n'est divisible que par lui-même et par l'unité.*" ... "On trouve de cette manière que les nombres premiers sont, 2, 3, 5, 7, ..."
no	1840	Lieber et al.	[84, p. 334]	[Encyclopædia Americana] "PRIME NUMBERS are those which have no divisors, or which cannot be divided into any number of equal integral parts, less than the number of units of which they are composed; such as 2, 3, 5, 7, 11, 13, 17, &c."
no	1842	R. C. Smith	[123, p. 118]	[Schoolbook] "A Prime Number is one that is divisible only by itself or unity, as 2, 3, 5, 7, 11, 13, 17, &c."
no	1844	J. Ozanam	[104, p. 16]	(non-mathematician!) "A *prime* number is that which has no other divisor but unity." Table of primes from 1 to 10,000 (same page) starts at 2.
no	1845	C. Beck	[8, p.77]	"*Pour décomposer un nombre en ses facteurs premiers, il faut le diviser successivement autant de fois que possible, et ensuite les quotients obtenus, par chacun des nombres premiers 2, 3, 5, 7, 11, etc., ...*"
yes	1848	J. B. Weigl	[135, p. 28]	[Schoolbook] "**Der ersten Primzahlen sind: 1, 2, 3, 5, 7, 11, 13, 15**[*sic*] ..."
yes	1853	E. Hinkley	[63, p. 7]	[Table, low level] Preface, page 3: "THIS is the first book, made or published in the country, devoted exclusively to the subjects of *prime numbers* and *prime factors*." Page 7: "The numbers 1, 2 and 3, are evidently prime numbers."
no	1854	P. L. Chebycheff	[23, p. 51]	Reprints Tchebycheff 1854 "Mémoire sur les nombres premiers" which states "Ce sont les questions sur la valeur numérique des séries, dont les termes sont des fonctions des nombres premiers 2, 3, 5, 7, 11, 13, 17, etc."
no	1854	C. J. Hargreave	[47, p. 34]	Glaisher cites: "On the law of prime numbers, Philosophical Magazine," Series 4, viii. (Aug. 1854), pp. 114-122.

prime	year	who	reference	quote (or comment)
yes	1854	A. Comte	[31, p. 420]	[Philosopher, non-mathematician] Mill [97, p. 196] writes "But M. Comte's puerile predilection for prime numbers almost passes belief. His reason is that they are the type of irreductibility : each of them is a kind of ultimate arithmetical fact. This, to any one who knows M. Comte in his later aspects, is amply sufficient. Nothing can exceed his delight in anything which says to the human mind, Thus far shalt thou go and no farther. If prime numbers are precious, doubly prime numbers are doubly so; meaning those which are not only themselves prime numbers, but the number which marks their place in the series of prime numbers is a prime number. Still greater is the dignity of trebly prime numbers; when the number marking the place of this second number is also prime. The number thirteen fulfils these conditions : it is a prime number, it is the seventh prime number, and seven is the fifth prime number. Accordingly he has an outrageous partiality to the number thirteen. Though one of the most inconvenient of all small numbers, he insists on introducing it everywhere."
no	1856	V. A. Lebesgue	[77]	[Not 'the' Lebesgue!] "... on représentera la suite complète des nombres premiers par $p_0 = 2, \ldots$"
yes	1859	V. A. Lebesgue	[78, p. 5]	"... les nombres premiers 1, 2, 3, 5, 7, 11, 13, ..."
yes	1862	V. A. Lebesgue	[79, p. 10]	"On extend par *diviseur* d'un nombre n tout nombre que s'y trouve contenu une ou plusiers fois exactement; quel que soit n, les nombres 1 et n en sont diviseurs. Le nombre n est *premier* lorsqu'il n'a que ces deux diviseurs; il est *composé* dans le cas contraire. Les nombres 1, 2, 3, 5, 7, 11, 13, 17, 19,... sont premiers;"
no	1863	L. Dirichlet	[34, p. 12]	[Note Dirichlet died 1859!] "Da jede Zahl sowohl durch die Einheit, als auch durch sich selbst theilbar ist, so hat jede Zahl – die Einheit selbst ausgenommen – mindestens zwei (positive) Divisoren. Jede Zahl nun, welche keine anderen als diese beiden Divisoren besitzt, heisst eine *Primzahl (numerus primus)*; es ist zweckmässig, die Einheit nicht zu den Primzahlen zu rechnen, weil manche Sätze über Primzahlen nicht für die Zahl 1 gültig bleiben." This last part is "It is convenient not to include unity among the primes, because many theorems about prime numbers do not hold for the number 1" [36, p. 8]. The parenthetical "(positive)" was not in the 1863 edition, but added by the 1879 edition [35, p. 12]. Nice quote: "Thus in a certain sense the prime numbers are the material from which all other numbers may be built" [36, p. 9].
yes	1863	J. Bertrand	[10, p. 342]	Definition [p. 86] "Un nombre entier est dit premier lorsqu'il n'a pas d'autres diviseurs entiers que lui-même et l'unité. EXEMPLES. 2, 3, 5, 7, sont des nombres premiers, 9 n'est pas premier, car il est divisible par 3." Despite this example, Table I: "Contenant tous les nombres premiers depuis 1 jusq'à 9907" starts, like it says, at 1 (pg. 342).

prime	year	who	reference	quote (or comment)
no	1864	V. A. Lebesgue	[80, p. 12]	The "TABLEAU des nombres premiers impairs, inférieurs à 5500" (page 2) lists 24 odd prime less than 100, starting at 3.
yes	1866	J. Ray	[109, p. 50]	[Schoolbook]
yes	1867	C. Aschenborn	[4, p. 86]	[Schoolbook for artillery and engineering]
no	1870	E. Meissel	[47, p. 34]	Glaisher cites: "Ueber die Bestimmung der Primzahlen innerhalb der ersten Hundert Millionen natürlichen Zahlen vorkommen," Mathematische Annalen, iii (1871), pp. 523-525.
yes	1870		[30, p. 131]	[Schoolbook, Rhode Island] "*Teacher.* Name all of the prime numbers from 1 to 50. *Pupil.* 1, 2, 3, 5, 7, . . . "
yes	1873	E. Brooks	[12, p. 58]	[Schoolbook] Mostly questions, no answers.
no	1875	G. M'Arthur	[88, p. 528]	[Encyclopædia Britannica, 9th ed.] Entry for Arithmetic: "A *prime number* is a number which no other, except unity, divides without a remainder; as 2, 3, 5, 7, 11, 13, 17, &c." Later an example: "The *prime factors* of a number are the prime numbers of which it is the continued product. Thus, 2, 3, 7 are the prime factors of 42; 2, 2, 3, 5, of 60."
yes	1876	M. Glaisher	[85, p. 232]	[Table] "M. GLAISHER, en comptant 1 et 2 comme premiers, a trouvé les valeurs suivantes: . . . " [M. Glaisher, by counting 1 and 2 as first, has found the following values: . . .]
yes	1876	K. Weierstrass	[134, p. 391]	"Dies führt zu dem Begriff der Primzahlen. Nimmt man die Primzahlen sämmtlich als positiv an, so kann man jede Zahl als Product von Primzahlen und einer Einheit +1 oder −1 darstellen, und zwar auf eine einzige Weise. Der Begriff der Primzahlen kann im Gebiete der complexen ganzen Zahlen, die aus den vier Einheiten 1, −1, i, $-i$ durch Addition zusammengesetzt sind, aufrecht erhalten werden. Denn jede Zahl $a + bi$ lässt sich auf eine einzige Weise durch ein Product von primären Primzahlen und einer der vier Einheiten ausdrücken."
yes	1877	M. de Mondesir	[47, p. 34]	Glaisher cites: "Compte Rendu"
no	1880	H. Scheffler	[117, p. 79]	"Hiernach sind die reellen Primzahlen 2, 3, 5, 7 . . . , welche früher dafür gehalten wurden, sämmtlich gemeine reelle Primzalen und . . . "
no	1887	G. Wertheim	[136, p. 20]	"Wir wollen die Anzahl der Zahlen des Gebiets von 1 bis n, welche durch keine der i ersten Primzahlen $p_1 = 2, p_2 = 3, p_3 = 5, \ldots, p_i$ theilbar sind, durch $\phi(n, i)$ bezeichnen."

prime	year	who	reference	quote (or comment)
no	1889	P. Chebyshev	[128, pp. 2–3]	"*Einfach* heisst eine Zahl, welche nur durch *Eins* und durch sich selbst theilbar ist; eine solche wird auch *Primzahl* genannt. *Eine zusammengesetzte Zahl* nennt man dagegen eine solche, welche durch eine andere Zahl, die grösser als *Eins* ist, ohne Rest getheilt werden kann. So sind 2, 3, 5, 7, 11, und viele andere *Primzahlen*, hingegen 4, 6, 8, 9, 10 und andere dergleichen *zusammengesetzte Zahlen*."
yes	1890	A. Cayley	[20, p. 615]	[Encyclopædia Britannica, 9th ed.] Entry for Number: "In the ordinary theory we have, in the first instance, positive integer numbers, the unit or unity 1, ..." "A number such as 2, 3, 5, 7, 11, &c., which is not a product of numbers, is said to be a prime number; and a number which is not prime is said to be composite. A number other than zero is thus either prime or composite; ..." "Some of these, 1, 2, 3, 5, 7, &c. are prime, others, $4,= 2^2, 6,=$ 2.3, &c., are composite; and we have the fundamental theorem that a composite number is expressible, and that in one way only, as a product of prime factors, $N = a^\alpha b^\beta c^\gamma \ldots (a, b, c, \ldots$ primes other than 1; $\alpha, \beta, \gamma, \ldots$ positive integers)."
no	1891	E. Lucas	[86, pp. 350]	"Il y a donc deux espèces d'entiers positifs, les nombres premiers et les nombres composés; mais on doit observer que l'unité ne rentre dans aucune de ces deux espèces et, dans la plupart des cas, il ne convient pas de considérer l'unité comme un nombre premier, parce que les propriétés des nombres premiers ne s'appliquent pas toujours au nombre 1." In a footnote he gives the example "Ainsi le nombre 1 est premier à lui-même, tandis qu'un nombre premier p n'est pas premier à lui-même; ..."
yes	1892	W. Milne	[98, pp. 91–92]	[Schoolbook] "Thus 1, 3, 5, 7, 11, 13, etc., are prime numbers." On page 95, 1 is not listed as a prime factor of 1008.
yes	1892	R. Fricke	[28, p. 592]	"Man bezeichne nun die Primzahlen 1, 2, 3, 5, ..."
no	1893	J. P. Gram	[50, p. 312]	For example, reports $\pi(100000) = 9592$ (which requires 1 be omitted) (he uses Θ instead of π).
no	1894	P. Bachmann	[5, p. 135]	"Denkt man sich sodann alle Primzahlen bis zu einer bestimmten Primzahl p hin, 2, 3, 5, 7, $\ldots p_0$, p, ..."
yes	1897	R. Frick & F. Klein	[42, p. 609]	"Die der Primzahl x voraufgehenden Primzahlen seien $1, 2, 3, 5, \ldots, \lambda$ so dass l ein Multiplum des Productes $2 \cdot 3 \cdot 5 \cdots \lambda$ ist."
yes	1901	L. Kronecker	[72, p. 303]	"...dafs die 16 Primzahlen 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47 kleiner sind als 50."
yes	1904	G. Chrystal	[25, p. 38]	[Advanced Schoolbook] "For example, 1, 2, 3, 5, 7, 11, 13, ...are all prime integers, ..." (defines composite first).

prime	year	who	reference	quote (or comment)
yes	1908+	G. H. Hardy	[54]	[First-year University Textbook] Hardy implies that 1 is prime in at least two places. First, while discussing Euclid's proof that there are infinitely many primes, Hardy notes [54, pp. 122] "If there are only a finite number of primes let them be 1, 2, 3, 5, 7, 11, . . . N." This was unchanged for the first six editions of his text 1908, 1914, 1921 [55, pp. 143-4], 1925, 1928 and 1933. (See the Hardy 1938 entry on page 18.) Next, he writes [54, pp. 147] "The decimal .111 010 100 010 10 . . ., in which the nth figure is 1 if n is prime, and zero otherwise, represents an irrational number." This example remained the same in all 10 editions (e.g., [55, pp. 174], and "the revised 10th edition" 2008 [58, p. 151]). He also has the ambiguous statement ([54, p. 48], [55, p. 56]) "Let y be defined as *the largest prime factor of* x (cf. Exs. VII. 6). Then y is defined only for integral values of x. When $x = \pm\ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, \ldots$ $y = \quad 1, 2, 3, 2, 5, 3, 7, 2, 3, \ \ 5, 11, \ \ 3, 13, \ldots$ The graph consists of a number of isolated points." This is essentially unchanged in the revised 10th edition [58, p. 151]; but whether or not 1 is considered prime, it is reasonable to accept 1 as the largest prime factor of 1. (Certainly 1 is the largest prime power dividing 1.)
no	1909	E. Landau	[76, p. 3]	[Euclid] "Unter einer Primzahl versteht man eine positive ganze Zahl, welche von 1 verschieden und nur durch 1 und durch sich selbst teilbar ist. Die Reihe der Primzahlen beginnt mit 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, . . ."
no?	1910	W. F. Sheppard	[120, p. 531]	[Encyclopædia Britannica, 11th ed.] Entry for Arithmetic: "A number (other than 1) which has no factor except itself is called a *prime number*, or, more briefly, a *prime*. Thus 2, 3, 5, 7 and 11 are primes, for each of these occurs twice only in the table. A number (other than 1) which is not a prime number is called a *composite* number." "The number 1 is usually included amongst the primes; but, if this is done, the last paragraph [talking about the fundamental theorem of arithmetic] requires modification, since 144 could be expressed as $1.\ 2^4.\ 3^2$, or as $1^2.\ 2^4.\ 3^2$, or as $1^p.\ 2^4.\ 3^2$, where p might be anything"
no	1910	G. B. Mathews	[91, p. 851]	[Encyclopædia Britannica, 11th ed.] Entry for number: "The first noteworthy classification of the natural numbers is into those which are prime and those which are composite. A prime number is one which is not exactly divisible by any number except itself and 1; all others are composite." That definition is ambiguous, but later on the same page to he clearly is excluding unity from the primes: "Every number may be uniquely expressed as a product of prime factors.

prime	year	who	reference	quote (or comment)
				Hence if $n = p^\alpha q^\beta r^\gamma \ldots$ is the representation of any number n as the product of powers of different primes, the divisors of n are the terms of the product $(1+p+p^2+\ldots+p^\alpha)(1+q+\ldots+q^\beta)(1+r+\ldots+r^\gamma)\ldots$ their number is $(\alpha+1)(\beta+1)(\gamma+1)\ldots$, and their sum is $\Pi(p^{\alpha+1}-1)\div\Pi(p-1)\ \ldots$;"
				Same article [91, p. 863] "... we examine Mersenne's numbers, which are those of the form 2^p-1, with p a prime; the known cases for which a Mersenne number is prime correspond to $p = 2, 3, 5, 7, 13, 17, 19, 31, 61$." [If 1 was prime, so would be 2^1-1.]
no	1912	H. v. Mangoldt	[87, p. 176]	"Ein anderes Biespiel ist die Reihe 2; 3; 5; 7; 11; $\cdots$ der Primzahlen."
yes	1914	D. N. Lehmer	[82]	[Table] Begins his introduction as follows: "A prime number is defined as one that is exactly divisible by no other number than itself and unity. The number 1 itself is to be considered as a prime according to this definition and has been listed as such in the table. Some mathematicians [a footnote here cites E. Landau [76]], however, prefer to exclude unity from the list of primes, thus obtaining a slight simplification in the statement of certain theorems. The same reasons would apply to exclude the number 2, which is the only even prime, and which appears as an exception in the statement of many theorems also. The number 1 is certainly not composite in the same sense as the number 6, and if it is ruled out of the list of primes it is necessary to create a particular class for this number alone."
no	1923	E. Hecke	[62, p. 5]	"Die Einheiten ±1 wollen wir nicht zu den Primzahl rechnen."
no	1929	G. H. Hardy	[56]	"More amusing examples are (c) $0.011010100010100\cdots$ (in which the 1's have prime rank) and (d) $0.23571113171923\cdots$ (formed by writing down the prime numbers in order)." [56, p. 784] (Example (c) is in all the editions of his *A Course of Pure Mathematics* where he included 1 as prime. To see this entry go to page 17.)
				In Euclid's infinitely many primes proof he states [56, p.802] "If the theorem is false, we may denote the primes by $2, 3, 5, \cdots, P$, and all numbers are divisible by one of these." [He also mentions this proof in his *A Course of Pure Mathematics* (1908) in which he mentions 1 is prime (see entry on page 17) and in his 7th edition in which again 1 is not listed as prime (see entry on page 18).
no	1938	G. H. Hardy	[57]	[First-year Textbook] While presenting Euclid's proof of the infinitude of primes Hardy writes [57, pp. 125] "Let $2, 3, 5, \ldots, p_N$ be all the primes up to $p_N, \ldots$" This is a change from the previous editions where 1 was prime (see the Hardy 1908 entry on page 17). This wording is used from the 7th edition (1938) through the revised 10th edition (2008).
yes	1942	M. Kraitchik	[71, p. 78]	[Expository] "For example, there are 26 prime numbers between 0 and 100, ..."

prime	year	who	reference	quote (or comment)
no	1949	van der Waerden	[129, p. 59]	"11. By prime numbers we usually understand on the positive prime numbers $\neq 1$, such as 2, 3, 5, 7, 11, ..."
yes	1964	A. Beiler	[9, p. 211]	[Expository] "From the humble 2, the only even prime, and 1, the smallest of the odd primes, they rise in an unending succession aloof and irrefrangible." (See also pp. 212–13, 223.)
no	1975	Shallit	[119]	J. Shallit, as a student, wrote an interesting note about the prime factorization of one suggesting that its prime factorization should be regarded as the empty list.
yes	1997	C. Sagan	[115, p. 76]	[Fiction] The aliens in the novel *Contact* transmit the first 261 primes starting 1, 2, 3, 5, 7, ...
yes	2011	Carnegie Lib. of Pittsburgh	[18, p. 13]	[Expository] "A prime number is one that is evenly divisible only by itself and 1. The integers 1, 2, 3, 5, 7, 11, 13, 17, and 19 are prime numbers. ...the largest known (and fortieth) prime [*sic*] number: $2^{20996011} - 1$. ...Mersenne primes occur where 2^{n-1} [*sic*] is prime."
yes	2012	Andreasen et al.	[2, p. 342]	[Schoolbook] "**prime number:** A number with exactly two whole number factors (1 and the number itself). The first few prime numbers are 1, 2, 3, 5, 7, 11, 13, and 17."

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